Buoyancy driven rotating boundary currents

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1. Introduction

Buoyancy is responsible for the formation of many important geophysical and astrophysical flows. In a rotating system, a sustained source of positive or negative buoyancy may give rise to an extended horizontal current against a lateral boundary, a gravity current. In such a current, the Coriolis force associated with the horizontal flow in the current is balanced by an offshore pressure gradient supported by the wall. The resulting flow is then bounded by a solid boundary on the right, (looking along the direction of flow in the Northern hemisphere) and by a front on the left. If the wall is sloping rather than purely vertical, the component of the Coriolis force parallel to the wall in the vertical plane can balance the similar component of the gravitational force acting on the current. Where the source is persistent, such currents can reach states of quasi-equilibrium. See Griffiths [1] for a review of boundary gravity currents.

It has been noted that the instability of many varieties of sheared flows cannot be determined locally [2]. As an illustrative example, recall that for a two-dimensional incompressible shear flow, $U = U(y)$, one *must* use boundary conditions appropriate to the corresponding eigenvalue problem in order to obtain a necessary condition for instability. That condition is simply the vanishing of the vorticity gradient ($U_{yy} = 0$) somewhere in the

flow – the celebrated Rayleigh criterion. Local approximation leads to a different condition and the erroneous prediction of likely instability [3].

Investigation of more complicated (compressible) 2-D shear flows has confirmed the nonlocality of wave-interaction, or resonant, instabilities [4] and more specific studies of boundary current configurations which account for the coupling of the current to its environment have also found that mode resonances can alter the intrinsic stability characteristics of flows [5,6,7].

2. The Meddy problem

How can the presence of warm, salty Mediterranean water be explained when it is found in the Western Atlantic ocean? The Mediterranean water is delivered in compact lens-shaped vortices affectionately known as *Meddies*. Such long-range mixing capabilities of some currents has a profound influence on the global ocean circulation. This example poses two distinct questions:

1 *How are Meddies formed?*

2 *How do they subsequently propagate ?*

The propagation of coherent patches of vorticity is only partially understood. A number of different long-range mechanisms can affect the motion of vortices. These include: interactions between different vortices, advection by a large scale background flow, interaction with a background vorticity or potential vorticity gradient (a good example of this is the "planetary beta" effect seen in flows in a thin shell of fluid on the surface of a rotating spherical planet [8]), interaction of a vortex with a horizontal or vertical boundary,

small scale dissipation within and around a vortex, and MHD effects in examples of astrophysical vortices.

3. Laboratory experiments

To investigate question 1, we developed a series of laboratory experiments to study buoyancy-driven currents in rotating stratified environments. Although we have targeted a specific oceanographic flow to model, it is tantalizing to consider that a reduced problem such as the one we propose here may also be relevant to other problems. In binary star systems, to consider one possible example, infall material from one star can feed an accretion disk around the other; the subsequent equilibration of the new material may well lead to buoyancy driven currents of a similar nature. But this study is primarily a model for the Mediterranean outflow, where we have the benefit of comparison with direct observations. The two principal goals of the experiment were:

(a) to determine the structure (velocity shear and thickness) of the currents initially formed from a negatively buoyant inflow

(b) to examine the stability properties of these current

A diagram of he experimental apparatus is shown in figure 1. The interface of the stable ambient two-layer stratification intersects the surface of a conical section at mid-depth. A thymol blue solution of intermediate density is held outside the tank and also fills a reservoir at the tank's outer edge. The densities of the overflow water (ρ_2) and the bottom layer (ρ_3) were achieved by dissolving sugar in distilled water.

A current is initiated by establishing a flow that causes overflow in the reservoir and the subsequent buoyant descent of the overflow water down the sloping surface toward the

layer interface, where it becomes neutrally buoyant. Because of the rotation, the descending current turns to flow cyclonically along the topographic slope. Except for extreme initial conditions, the current will still reach the layer interface. Along one radius of the conical surface, an array of fine (0.006" diameter) vertical wires are placed at 0.44 cm intervals. The wires are pulsed with an electrical current to activate thymol blue tracer in fluid of the current, allowing direct visualization. A camera is positioned above the tank (where the length of advected tracer is used to determine velocities) and a mirror is oriented to allow a head-on view of the current (where the height of unadvected tracer determines local thickness). Crude evaluation of the response in the ambient layers is done by tracking floating and, where possible, submerged tracers. Images captured with the CCD camera were digitally stored using a video framer and the measurement of tracer was facilitated by enhancing contrast using standard algorithms. Pre-marked intervals on the experimental surfaces provided accurate distance standards, and the real-time record kept on video simplified measurement of time intervals.

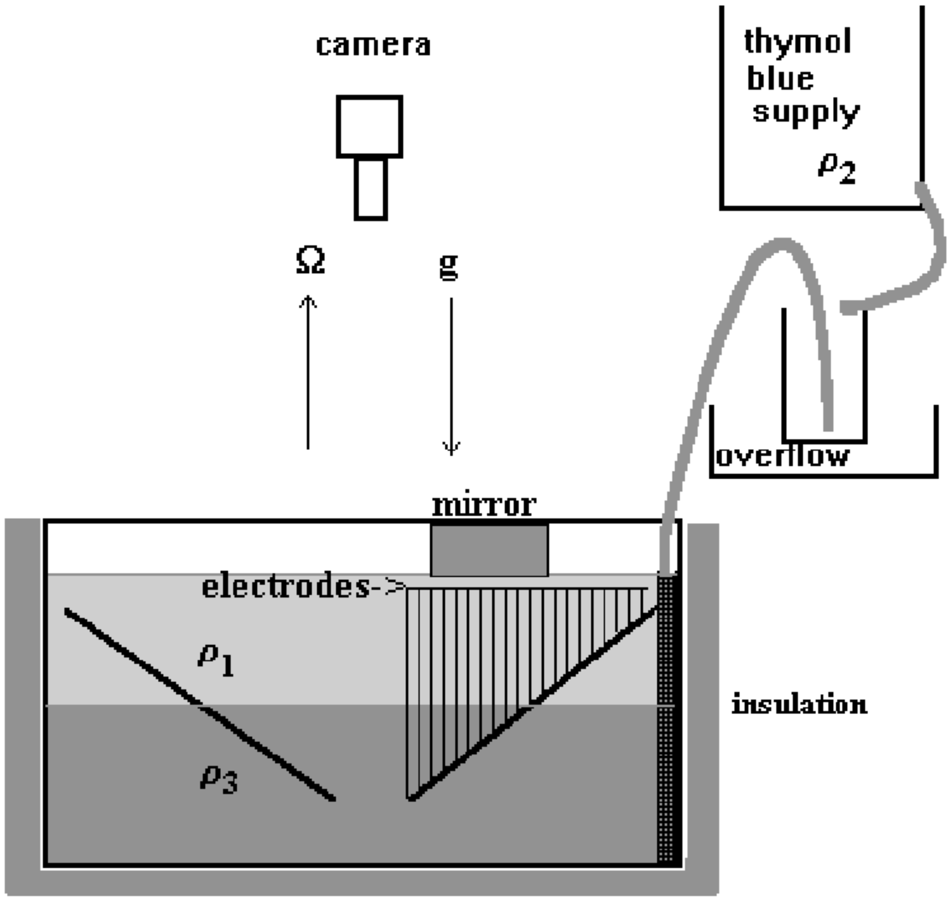

Figure 1. The experimental apparatus: the cylindrical tank and the submerged conic slope are shown in cross-section; the tank has diameter of $\sim$ 60 cm. Everything sits on a rotating table.

In all experiments, the densities were prepared with the values: $\rho_1 = 1.0$g/cc, $\rho_2 = 1.005$g/cc, and $\rho_3 = 1.01$g/cc; the depths of the ambient layers were 11cm each. The reduced gravity between layers then took the values: $g'_{12} = g'_{23} = 5.0$g cm/sec^2; while Ω was either 1. or 1.5. The principal control value was the volume flux, $\mathcal{S}$, of ρ_2-fluid, which was varied from $0.15 - 1.0$cc/sec. $\mathcal{S}$ is directly related to a characteristic current depth H, which varied from $1 - 3$cm. Two values of the topographic slope, α, were used, 0.0 (a

vertical wall) and 0.25.

From the parameters above, we can construct several useful quantities. A Rossby deformation length is given by $L_D = (g'H)^{\frac{1}{2}}/2\Omega$, and the ratio of the current width to L_D is indicative of the relative importance of baroclinic and barotropic disturbances. A Reynolds number is given by $Re = (g'H^3/\nu^2)^{\frac{1}{2}}$, and for these experiments reached nearly 10^3, but a more useful quantity is the Froude number $F = U^2/(g'H)$.

These and other experiments have verified that baroclinic currents and instabilities of interest are characterized not so much by Re as by F. In oceanic flows, F can vary between 10^{-2} and 10^2, and in the laboratory we can easily produce a range of F between 10^{-2} and 10, simply by forming velocities U in the range 0.1 to 1 cm/sec.

In high Reynolds number flows without ambient stratification, experiments have shown that instabilities of Kelvin-Helmholtz type typically grow rapidly at the nose, while further upstream – where the steady current is well established – baroclinic instabilities produce eddies. We focus on the established current rather than its nose and the details of the intrusion process. For this reason, we initiate the current with a small density enhancement to minimize the initial Kelvin-Helmholtz mixing, while using a tank of sufficient horizontal dimension to traverse the relevant Froude number range.

4. Steady three layer currents

What is the three-layer, steady, streamwise uniform solution? An arbitrary three layer arrangement of fluid is described by the familiar shallow layer system:

$$\frac{d\mathbf{u}_1}{dt} + \mathbf{f} \times \mathbf{u}_1 = -g\nabla(h_1 + h_2 + h_3) \;, \tag{1}$$

$$\frac{d\mathbf{u}_2}{dt} + \mathbf{f} \times \mathbf{u}_2 = -g\frac{\rho_1}{\rho_2}\nabla h_1 - g\nabla(h_2 + h_3) \;, \tag{2}$$

$$\frac{d\mathbf{u}_3}{dt} + \mathbf{f} \times \mathbf{u}_3 = -g\frac{\rho_1}{\rho_3}\nabla h_1 - g\frac{\rho_2}{\rho_3}\nabla h_2 - g\nabla h_3 \ , \tag{3}$$

and

$$\frac{\partial h_i}{\partial t} + \nabla \cdot (h_i \mathbf{u}_i) = 0 \ , \tag{4}$$

holds for each layer $i = 1, 2, 3$.

We will let Ox be the streamwise direction, parallel to the boundary, and Oy, the cross-stream direction. For a boundary current of intermediate density ($i = 2$) flowing against a vertical wall, a steady, streamwise uniform current with velocity $\bar{u}(y)$ and thickness $\bar{h}(y)$ must satisfy potential vorticity and momentum conservation.:

$$\frac{d}{dt}\left(\frac{f - \partial_y \bar{u}}{\bar{h}}\right) = 0 \quad \& \quad f\bar{u} = -g^* \partial_y \bar{h} \tag{5}$$

where the reduced gravity here is $g^* = g\frac{(\rho_2-\rho_1)(\rho_3-\rho_2)}{\rho_2(\rho_3-\rho_1)}$.

For uniform *non-zero* initial potential vorticity, $Q_2(t=0) = Q_*$, we find currents of width L having:

$$\bar{u}(y) = fl\frac{\sinh(y/l)}{\cosh(L/l)} \quad \& \quad \bar{h}(y) = h_* - h_*\frac{\cosh(y/l)}{\cosh(L/l)} \ , \tag{6}$$

where a deformation length is now associated with the initial conditions:

$$l = \left(\frac{g^*}{fQ_*}\right)^{1/2} \ . \tag{7}$$

To extend the solutions above to the laboratory configuration, in which the boundary along which the current flows has some slope, we must simply match the solution above to a two layer solution of a current flowing along a slope (this sub-problem is the subject of several previous studies, see the review [1] for references).

5. Results

Steady solutions are shown in figure 2 alongside measured profiles. Experiments performed with vertical boundaries have found the strong tendency of currents to be unstable, even when the currents were initiated from a uniform potential vorticity source. This instability set in uniformly along the current once it became sufficiently wide, forming a chain of eddies. The eddies were roughly the same width as the current and moved irregularly following separation.

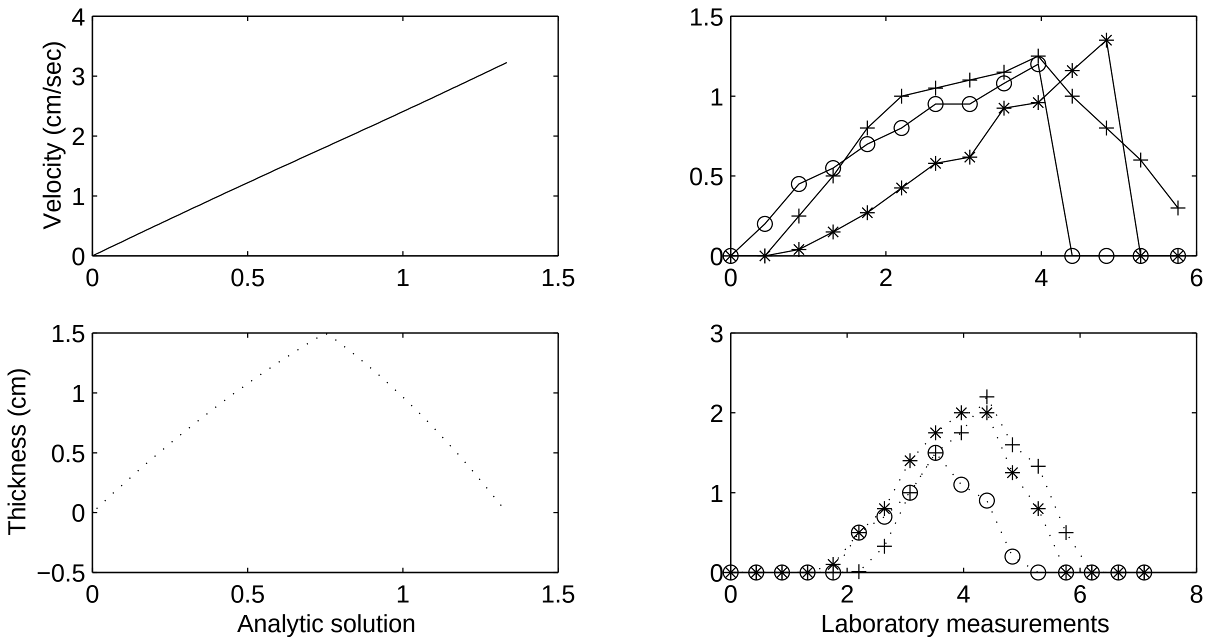

Figure 2. (left) Steady solutions for u-velocity and thickness of the boundary current in the three-layer system flowing along a sloping boundary; (right) The measured u-velocities and thickness for three experiments, one case conincides with the shedding of a cyclone.

Analogous experiments along sloping boundaries have found the currents to be stable over much longer times and for wider currents. The stabilization of such currents by the presence of a slope should be emphasized – it suggests that the instabilities found for highly idealized and often symmetrical posed problems may not be realized in more general settings. The first step in proving or disproving this hypothesis is understanding the steady

current configurations and a subsequent stability analysis. What we have reported here are first measurements of steady current profiles for these stable cases.

Related experiments, in which a surface current is allowed to eventually encounter a sloping boundary, have also found that a slope can have a stablizing effect – following the encounter, the current was found to be slower, wider, and more laminar than it would be along a vertical wall[9].

A final point concerns the formation of Meddies: in the process of initiating experimental currents, the intermediate density fluid formed two distinct equilibria. The first as it flowed through less dense water, down the slope and to the right; after reaching the layer interface with the densest water, it spread to the right and along the sloping boundary. Both stages were rather laminar and steady, but at the transition, a single large cyclone was typically shed. The Mediterranean undergoes a similar transition where it negotiates a sharp turn at Cape St. Vincent and presumably other significant topographic variations, such as channels, are also encountered. The role of such localized perturbations may help explain why, when experimental currents suggest stability, their oceanic counterparts are able to form eddies.

The excitation of inertial waves in the ambient homogeneous fluid is also found.

Finally, we would like to use these preliminary results to initiate a more thorough investigation of the linear instability in the full three layer problem as well as the basis for a companion set of numerical experiments to duplicate those done in the laboratory.

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